COVER SHEET

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RAFTERY CURVES FOR TENDER PRICE FORECASTING: EMPIRICAL PROBABILITIES AND POOLING

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RAFTERY CURVES FOR TENDER PRICE FORECASTING: EMPIRICAL PROBABILITIES AND POOLING

ABSTRACT

A method is proposed for the empirical derivation of Raftery Curve probabilities from forecasted/actual value ratios. The method is applied to a set of Hong Kong construction contract data. Using the error of predicted ratios as the measure of opportunity cost, it is then shown how the method may be used to identify the best data poolings amongst subsets.

Keywords: Raftery Curves, tender price forecasts, empirical probabilities, data pooling, Hong Kong construction contracts.

INTRODUCTION

Estimates of construction costs are potentially more useful in probabilistic form instead of the conventional point estimate. For estimates obtained by synthesising construction activity costs, the Program Review and Evaluation Technique (PERT) provides a means of combining probabilistic activity estimates in the form of means and ranges. This has been developed into what has become known as 'range estimating' (eg., Knoke et al, 1993, Rowland and Curran, 1995; Curran, 1995, Curran and Curran, 1996; Isidore and Back, 1999; Back, 2001). Raftery (1993) has suggested that this could usefully be developed further to show the form (shape) of the distribution (termed Raftery Curve) as well as the range. Skitmore (1996) has shown how this might be done parametrically for tender price forecasts, by fitting one of the standard probability density functions to the cumulative frequency of the tender price forecast/actual values.

In this paper, a method is developed for constructing empirical (nonparametric) Raftery Curves for tender price forecasts. This is applied to a set of Hong Kong construction contract data. Using the error of predicting ratios as the measure of opportunity cost, it is then shown how the method may be used to identify the best data poolings amongst subsets.

RAFTERY CURVE CONSTRUCTION

In its parametric form, Raftery Curve construction involves two major considerations (1) the appropriate form of probability density function (pdf), such as the normal, lognormal, uniform or beta distribution and (2) the parameter estimates of the pdf, such as the mean and variance of the normal distribution or the minimum and maximum of the uniform distribution. In the tender price forecasting field, most of the empirical research to date has concentrated on (2). Ashworth and Skitmore (1983), Skitmore (1991) and Gunner and Skitmore (1999) have provided summaries of many of these previous studies in terms of bias and consistency - bias being the average of differences between actual bid prices (usually lowest tender price) and forecasts, and consistency being the degree of variation around this average. The only tests for an appropriate pdf have been on the normal distribution as a check on the reasonableness of assumptions involved in regression analyses (eg., Gunner and Skitmore, 1999).

Also of relevance is the extensive research that has been carried out in the development of bidding models. In this case, as the construction contract bidders are trying to provide the lowest bid (tender), each bid can be assumed to be equivalent to a tender price forecast. Most commonly, the total bid price for each tenderer has been treated as a random variable from some well-known density function such as uniform (e.g. Cauwelaert and Heynig, 1978, Fine and Hackemar, 1970, Grinyer and Whittaker, 1973, Whittaker, 1970), normal (Cauwelaert and Heynig, 1978, McCaffer, 1976, Mitchell, 1977, Morrison and Stevens, 1980), lognormal (Brown, 1966, Klein, 1976, Skitmore, 1991, Weverbergh, 1982); gamma (Friedman, 1956), Weibull (Oren and Rothkopf, 1975) or just “positively skewed” (Beeston, 1974, McCaffer and Pettitt, 1976, Park, 1966). Often, tenderers are assumed to be homogeneous, which allows all bids to be regarded as coming from the same pdf. Very little of this work has involved model fitting though. McCaffer and Pettitt (1976), assuming homogeneity, found a normal distribution to be a better fit for their data than a uniform distribution. Skitmore (1991), however, found his data to be heterogeneous, with individual bidders following a three-parameter lognormal distribution. More recently, Skitmore (2001), assuming homogeneity, has fitted a two parameter lognormal (and normal) pdf to bidding data with high outliers removed.

A further issue concerns the data sampling used for curve fitting. To objectively construct a Raftery Curve for a future contract, the constructor is compelled to use a set of data for a sample of completed contracts. Beeston (1974) has urged the use of as large a sample as possible for analysis in order to reduce the effects of sampling bias. However, as originally pointed out by Flanagan (1980), this produces a paradoxical situation. Ideally, the constructor would use a sample of similar contracts to the future contract, ie., of similar functional and technological type, size, geographical location, etc. The assumption is that the closer the characteristics of the sample match the future contract, the better the ensuing Raftery Curve will be. However, the closer the sample is made to match the future contract, the smaller the sample becomes, and the greater become the sampling bias involved in curve fitting. Clearly, the solution to this dilemma is to somehow trade-off the biases created by using too small a sample with the biases created by using an unrepresentative sample.

AN EMPIRICAL APPROACH

Empirical probabilities are derived by rank ordering the forecast/tender price ratios and assigning the probability, p_i, to the ith ratio, r_i (i=1,...,n):

$$pi = \frac{i}{n} - \frac{i}{2n} \qquad (1)$$

Fig 1 shows a step plot of the results of this operation applied to a subset of small Hong Kong building contracts (see Appendix A for details of the whole dataset). By joining the points with a straight line, the probability for any ratio can be estimated by interpolation (extrapolation at the ends of the curve) between the two straddling points. Thus, the associated probability for a ratio of 0.90 is interpolated as around 0.054 (Fig 2). Similarly, the ratio can be estimated for a given probability also by interpolation/extrapolation, a probability of 0.05 occurring where the ratio value is approximately 0.89.

THE POOLING PROBLEM

Within-pooling opportunity cost

The pooling problem arises because the sample sizes are finite. Clearly, with an infinite sample size of identical contracts, the empirical probabilities will be exact for all ratio values. As the sample size reduces, the average gap between each observation increases. One way to measure the effect of this is to leave out an observation and see how the predictions of the model change. Fig 2 shows the situation. Here we leave out the third lowest ratio, which has a value of 0.9721 (p=0.0625). Rank ordering the remaining n-1 ratios and recalculating the probabilities produces the result shown in Fig 3. This shows the new curve marked with blobs, in contrast with the original curve marked with stars. The probability of 0.0625 now points to an interpolated ratio of 0.9488, a difference of 0.9721-0.9488=0.0216.

Now in practice, it is likely that the Raftery Curve user will want to know the value above or below the forecast that coincides some probability. For instance, a client might want to know beyond what percentage above or below the forecast there is little or no chance (eg., 1 in 20 or a probability of 0.05) of a bid occurring. The identical situation occurs when a bidder is interested in knowing below what percentage of the (cost estimate) forecast there is little or no chance of the lowest bid occurring. The point of interest in both situations is not so much how good is the probability prediction for a given percentage deviation from the forecast, but how good is the predicted percentage deviation for a given probability. Also, the goodness of the percentage deviation prediction is an important issue for the client or bidder. Let us say that, for both client and bidder, they need to know the percentage deviation from the forecast that accords with a probability of 0.05, ie., there is a probability of 0.05 the lowest bid will be less than the percentage stated. Imagine this percentage is predicted to be 2 percent. The client allows for this in the annual budget for all projects in the form of the expected saving 2x0.05=0.10 percent. Now, if the ***true*** percentage for a probability of 0.05 is actually 10 instead of 2 percent, this means that the expected saving should have been 10x0.05=0.50 percent instead of 0.10 percent. For large client organisations, this can result in a considerable dollar underspend. The same argument applies in the reverse case where an overspend may result. For bidders, such poor predictions can result in contracts lost (or worse) through applying excessively high mark-up values or the acquisition unprofitable contracts by bidding too low. Therefore, the difference between the predicted and actual percentage deviation from the forecast, for a given probability, represents an opportunity cost for both client and bidder users. Whether this opportunity cost is symmetrical (ie., the pain is the same for a given overspend as it is for the same amount of underspend) and a linear function of the goodness of the percentage deviation prediction (ie, twice the overspend means twice the pain) is an empirical issue yet to be determined (and may depend on the individual circumstances involved). It does seem, however, reasonable to assume at this stage a one to one correspondence between opportunity cost and the error of the deviation prediction.

In terms of the analysis so far then, the difference between the original ratio value for observation 3 and its predicted value for the same probability once it is omitted from the dataset is the opportunity loss. That is, the difference 0.0216 ***is*** the opportunity loss. Now, as we have assumed symmetry, there is no need to distinguish between positive and negative values of these differences. We can, therefore, simply sum the absolute (unsigned) values of these differences for the whole of the dataset as we continue to omit each observation in turn (with replacement). Table 1 summarises the results of doing this for each of the 40 small cfa valued contracts. The first column gives the contract sequence number, the second the ratio (*R*), the third the associated probability (*p*), the fourth is the predicted ratio for that *p* value once that contract is removed, the sixth column gives the difference between the original *R* and the predicted *R* and the last column gives its absolute value. Thus, contract 13, which is the third of the small cfa contracts shows the results in the above example. The last three rows provide the totals, means and standard deviations for the column values. Thus for the 40 contracts involved in this subgroup, the mean value of the ratios is 1.1474 - indicating that these contracts have been overestimated by an average of 14.74 percent - with a standard deviation of 16.48 percent (a coefficient of variation of 14.36 percent), which is normal for this type of forecast. The probabilities, as expected by virtue of the method of calculation, have a mean of 0.50. The mean value of the predicted ratios is, at 1.1432, slightly less than the mean of the actual ratios, indicating a small amount of bias in the predictions. With a standard deviation of 0.1328, the predicted ratios have quite a large reduction in variability than the original ratios, but again this is to be expected with the smoothing effect caused by the omission of observations. The mean difference between predicted and actual ratios is shown although this is calculable from the means of actual and predicted ratios (1.1474-1.1432=0.0042), but the standard deviation of the difference may be a useful measure as an alternative to the absolute differences in the event of the true opportunity costs involved turning out to a squared, rather than linear, function of the actual-predicted ratio differences. The mean of the last column, 0.0190, provides us with our current measure of opportunity loss for this data grouping.

This statistic can easily be computed for any pooling arrangement, provided all the data for the subgroup are present. For example, the results for the combined small, medium and large cfa groupings are given in Table 2. In this case, ***all*** the ratios are rank ordered and assigned probability values (only those for small cfa contracts are shown in Table 2). Each contracts is again left out in turn and ***all*** the remaining contracts reassigned probability values, which are then interpolated to obtain the predicted ratio values as before (again, only the predicted ratios for the small cfa contracts are shown in Table 2). From this the signed and unsigned differences are derived and analysed as before. The summary statistics in the last three columns show the changes produced by the pooling. In particular, we are interested in the mean absolute difference between the actual and predicted ratios. This is 0.0110 in contrast with the 0.0190 recorded for the unpooled data and represents a considerable reduction in opportunity cost. The results for all the possible poolings are summarised in Table 4. This shows the full pooling, as expected, to have the least opportunity cost (0.0110), followed by the small and medium cfa pooling (0.0115) the small and large cfa pooling (0.0153) and finally the unpooled version (0.0190).

Between-pooling opportunity costs

Different pooling schemes produce different ratio probabilities. Fig 4 illustrates this. The empirical Raftery Curves for the small cfa contracts are shown based on both the unpooled small cfa contracts and the fully pooled contracts. For convenience, we shall call these the *unpooled curve* and *pooled curve* respectively. Which curve provides the correct probabilities? Can both be correct?

The answer to this is that both are correct in their own way. The pooled curve is produced by all the data and is therefore appropriate if, for example, the cfa category for the future contract is not known. It follows, therefore, that the unpooled curve will be appropriate when the cfa category is known for the future contract. In practice, it is expected that the latter will be the case and therefore the unpooled curve is the correct one for the purpose. This means then that, for a given probability, the ratio predicted by the pooled curve is in error by the amount of its difference with the equivalent ratio for the unpooled curve. For example, the third ranked ratio (contract 13) for the pooled cfa subgroups is 0.9271, with a probability of 0.0620 (Table 2). Now, looking at Table 1, it can be seen that this probability is equivalent to a predicted ratio for the unpooled curve of between 0.8466 (p=0.0375) and 0.9271 (p=0.0625). By linear interpolation, this is a predicted ratio of 0.9255 - an error of 0.9271-0.9255 = 0.0017 (rounded). Table 3 columns {3} and {4} shows the results for all the small cfa contracts. Now, again assuming that the opportunity costs are a symmetrical and lineal function of this error, we can sum the absolute values of the errors over all the ratios in the subset to obtain the opportunity cost involved in using the pooled curve instead of the unpooled curve. In this example, the total is shown at the bottom of Table 3 column {4} as 1.7912, with a mean of 0.0448. This is the 'between-pooling' opportunity cost.

Total opportunity cost

Clearly, the total opportunity cost of using the pooled curve instead of the unpooled curve is the sum of the extra opportunity costs involved. These comprise (1) the difference between the within-pooling costs and (2) the between-pooling costs. These are shown in Table 3. Column {2} provides the difference between the within pooling costs and column {4} provides the between pooling costs. For example, the third small cfa contract (contract 13) has an unpooled within-pooling cost of 0.0216 (Table 1 last column) and a pooled within-pooling cost of 0.0265 (Table 2 last column) - a difference of 0.0048 (rounded) (Table 3 column {2}). To this is added the between-pooling cost of 0.0017 (Table 3 column {4}) to give the total opportunity cost of 0.0065 (Table 3 column {5}). The bottom of Table 3 column {5} then gives the total opportunity cost for the curve of 1.4719 (mean 0.0368) - indicating the extra cost involved in using the pooled curve instead of the unpooled curve. Therefore, in this case, when constructing an empirical Raftery Curve for a future small cfa contract it will be better use the unpooled data than to pool all the data.

ALTERNATIVE POOLINGS

The example so far has compared the unpooled small cfa dataset with the fully pooled cfa dataset. This is easily extended to other comparisons, in fact all combinations of subsets. For the cfa example, this involves a total of four such combinations - small only, small-medium, small-large and small-medium-large. Table 5 summarises the results for the small cfa contracts. Obviously, for the small cfa subset, the analysis is just comparing subset 1 with itself and there can be no change in opportunity cost. The result for the all combined subsets 1, 2 and 3 is identical to Table 3, with a mean opportunity cost of 0.0368. For the other poolings, the least opportunity cost is provided by the small-large pooling, (mean 0.0243) followed by the small-medium pooling (mean 0.0308). Thus, the best curve to use for a future small cfa contract with these data is the unpooled curve.

The same approach can be used for any other characteristic of the future contract (1) that is known in advance and (2) for which historical data is available. Table 6 summarises the results of a few of these for the Hong Kong dataset. For the remaining cfa categories, the analysis indicates that the large cfa contracts should be pooled (-0.0006 mean opportunity cost) but the medium cfa contracts should not be pooled. For the various construction types, type 1, 4, 5 and 7 are recommended not to be pooled, while type 3 contracts are recommended to be pooled with types 1, 2 and 7 (-0.0098 mean opportunity cost); type 8 contracts pooled with types 1, 2 and 6 (-0.0006 mean opportunity cost); and type 9 contracts pooled with type 3 contracts (-0.0354 mean opportunity cost). The groups, number of bids, nature and estimate value results are all interpreted the same way.

CONCLUSIONS

This paper has proposed a purely empirical method for constructing Raftery Curves for tender price forecasting and shown how this can be used to decide on an appropriate data pooling scheme. There are some difficulties, however, that need to be address. In summary these are:

1. The probability values, and linear interpolations, for the unpooled curve are assumed to be exact. This is clearly not the case with finite sample sizes. To illustrate the point, consider the situation where an unbiased die is thrown once and its value, say six, is recorded. Now the purely empirical probability of getting a six is unity, which cannot be correct as the true probability must surely be one sixth. By throwing the die many more times the empirical probability is going to close asymptotically on the true value but two are unlikely to coincide for a very long time. As a method of deciding subset-pooling arrangements, therefore, it is biased in favour of the unpooled subset.

2. As the data tends to be sigmoidal, the frequency distribution tails are long and therefore more error prone. Averaging errors over the whole curve therefore can be misleading as the results can be biased by one poor tail observation. One possible solution to this would be to restrict the averaging to a range of probabilities, say 0.05 to 0.10, within the tails. This may well suit the

requirements of the curve user too, as the interest is likely to focus on a small threshold range of probabilities such as these.

3. The method is described in univariate form. Although the method in theory can be extended to subgroup interactions, eg., small industrial buildings or low valued buildings with a small number of tenderers, by physically splitting the data accordingly, to do so would involve a rather large computation load and this, together with the inbuilt bias towards unpooled samples, suggests against further complication.

4. It is assumed that no other variables have a significant effect. For example, the data is assumed to be static, ie., it is assumed that no changes take place over time. This can, of course, be corrected by adding in whatever other significant predictor variables exist - providing they are known and measurable.

5. The error in predicting ratios is assumed to a suitable proxy for opportunity cost. It is also assumed to be symmetrical and linear. This can easily be tested empirically.

Clearly some form of smoothing should help ameliorate the difficulties in 1 and 2. The use of parametric methods, instead of the nonparametric method used here, offers one way of doing this and should also help with 3 and 4. This will involve some different assumptions though, particularly concerning the appropriate pdf to use.

REFERENCES

Ashworth, A., Skitmore, R.M., 1983, *Accuracy in estimating*, Occasional Paper No. 27, Englemere, Chartered Institute of Building, ISBN 0 906600 57 X.

Back, W.E., 2001, Integrated range estimating and stochastic scheduling. *Cost Engineering* **43**(2) 41-4.

Beeston, D.T., 1974, *One statistician's view of estimating*, *Cost Study F3*, London: Building Cost Information Service.

Brown, K.C., 1966. A theoretical and statistical study of decision making under uncertainty - a competitive bidding for leases of offshore petroleum tracts, PhD dissertation, Southern Methodist University, Dallas.

Cauwelaert, F.V, Heynig, E., 1978. Correction of bidding errors; the Belgian solution, *Journal of the Construction Division*, American Society of Civil Engineers **105**(CO1): 13-23.

Curran, K.M., Risk estimating adds value to value engineering. *Computing in Civil Engineering Congress* **2** 1212-9.

Curran, M.W., Curran, K.M., 1996. Range estimating. *Cost engineering* **26** 317-32.

Fine, B., Hackemar, G., 1970, Estimating and bidding strategy, *Building Technology and Management* Sep: 8-9.

Flanagan, R., 1980, Tender price and time prediction for construction works, PhD thesis, University of Aston in Birmingham.

Friedman, L, 1956, A competitive bidding strategy, *Operations Research* **1**(4): 104-12.

Grinyer, P.H., Whittaker, J.D., 1973, Managerial judgement in a competitive bidding model, *Operational Research Quarterly* **24**(2): 181-191

Gunner, J., Skitmore, M., 1999, Comparative analysis of pre-bid forecasting of building prices based on Singapore data. *Construction Management and Economics* **17**(5) 635-46.

Isidore, L.J., Back, W.E., 1999, Integrated range estimating and stochastic scheduling. *Annual Meeting AACE International* **43** RISK.03.

Klein, J.D., 1976, Joint ventures and bidding for offshore oil, in Amihud Y (ed), *Bidding and auctioning for procurement and allocation*, New York University Press.

Knoke, J.R., Liu, J.R., Spittler, J.R., 1993. Square foot range estimating. *Annual Meeting - AACE* **37** L.6.

McCaffer, R., 1976, Contractor's bidding behaviour and tender price prediction, PhD thesis, Loughborough University of Technology.

McCaffer, R., Pettitt, A.N., 1976, Distribution of bids for buildings and roads contracts, *Operational Research Quarterly*, **27**(4i) 835-43.

Mitchell, M.S., 1977, The probability of being the lowest bidder, *Applied Statistics* **2**(2): 191-94.

Morrison, N., Stevens, S., 1980, *Construction cost data base* 2nd annual report of research project by Dept of Construction Management, University of Reading, for Property Services Agency, Directorate of Quantity Surveying Services, DOE, UK

Oren, S. S., Rothkopf, M.H., 1975, Optimal bidding in sequential auctions, *Operations Research* **23**(6)

Park, W.R., 1966, The strategy of contracting for a profit, Prentice-Hall.

Raftery, J., 1993, Construction: perspectives from economics and psychology, Inaugural Lecture, the University of Greenwich, 16 Dec, School of Land Construction & Construction Management, Dartford Campus, Oakfield Lane, Dartford DA1 2SZ.

Rowland, W.P., Curran, K.M., 1995. Range estimating in value engineering: a risk reduction tool. *Value World* **18**(2) 25-9.

Skitmore, R.M., 1991, The contract bidder homogeneity assumption: an empirical analysis, *Construction Management and Economics* **9**(5) 403-29.

Skitmore, R.M., 1996. The Raftery Curve, *Proceedings*, Int Symp, The International Council for Building Research Studies and Documentation CIB W-55, 4-7 Sep, M Katavic (ed), Faculty of Civil Engineering, University of Zagreb, Kaciceva, Croatia, ISBN 953 96245 2 5, 241-52.

Skitmore, R.M., 2001, Graphical method for identifying high outliers in construction contract auctions. *Journal of the Operations Research Society* (in press)

Skitmore, R.M., Pemberton, J., 1994, A multivariate approach to contract bidding mark-up strategies, *J Opl Res Soc*, **45**(11) 1263-72.

Weverbergh, M., 1982, Competitive bidding: estimating the joint distribution of bids, Working paper 82-79, Dec, Centrum voor Bedrijfseconomie en Bedrijfseconometrie Universiteit Antwerpen - USFIA.

Whittaker, J.D., 1970, A study of competitive bidding with particular reference to the construction industry, PhD thesis, City University, London.

APPENDIX A: DESCRIPTION OF HONG KONG DATASET

The analysis is based on a sample of 229 projects with 3,285 bids received over the period from the fourth quarter of 1990 to the third quarter of 1996. The sample was derived from HKSAR Government Architectural Services Department (ASD). Projects awarded through selective tendering (where the number of competitors is an administrative decision rather than a consequence of market conditions) have been excluded. The bid value was calculated as the original bid value x 885 (TPI as at the fourth quarter of 1996), the TPI at tender opening date. Other details were also recorded for each project. Where not naturally grouped already, these were placed into roughly the same size groups of 'small', 'medium' and 'large' for analysis as shown below.

Nature of Work:
1 = New works
2 = Alteration works
3 = Maintenance works

Groups invited to tender:
The Government uses open tendering for the vast majority of its projects. For those projects that are large and/or complex, the Government uses selective tendering. The Hong Kong SAR Government maintains two lists of approved contractors. These are split into local and overseas. The local list is subdivided into three classes of contractor i.e. (1) Class A local contractors who can bid for small projects up to around HK $15 million, (2) Class B local contractors who can bid for small and medium projects up to around HK$50 million, (3) Class C local contractors who can bid for small, medium and large projects of unlimited value. The overseas contractors are listed separately on List II. They can only bid for large projects over $50 million. Newly listed contractors need to go serve a probationary period. An explanation of the groups invited to tender coding is as follows:
1 = A and above. This means that Class A, B and C local contractors can bid for the job
2 = A (excluding probationary status) and above
3 = B and above. This means that Class B and C local contractors can bid for the job
4 = B (excluding probationary status) and above
5 = C and above. This means that only Class C contractors can bid for the job.
6 = C (excluding probationary status)
7 = List II. This means that only overseas contractors can bid for the job.
8 = A (excluding probationary status) and Above and List II
9 = B and Above and List II
10 = B (excluding probationary status) and Above and List II
11 = C and List II
12 = C (excluding probationary status) and List II
13 = All listed tenderers
14 = Prequalified tenderers . This essentially means that selective has been used.

Number of bidders.
Small = less than 9 bidders
Medium = 9 to 15 bidders

Large = over 15 bidders

Estimate value
Small = less than HK$45,367,000
Medium = HK$45,567,00 to HK$100,368,000
Large = over HK$100,368,000

Construction type
1 = Utilities and civil engineering facilities
2 = Industrial facilities
3 = Administrative and commercial facilities
4 = Health and welfare facilities
5 = Recreational facilities
6 = Religious facilities
7 = Educational, scientific and information facilities
8 = Residential facilities
9 = Other facilities

Floor area
This refers to new works projects only. Hong Kong uses construction floor area (CFA) rather than gross floor area (GFA). CFA is measured to the outside face of the column.
Small = less than 4860 m2
Medium = 4860 m2 to 10658m2
Large = over 10658m2

CAPTIONS:

Fig 1: Cumulative frequency of small cfa contracts
Fig 2: Probability values
Fig 3: Probability values
Fig 4: Cfa probabilities

Table 1: Results for small cfa contracts subset only
Table 2: Results for all cfa subsets
Table 3: Comparison of the two poolings
Table 4: Summary of poolings
Table 5: Summary of comparisons for small cfa contracts
Table 6: Best poolings

Fig 1: Cumulative frequency of small cfa contracts

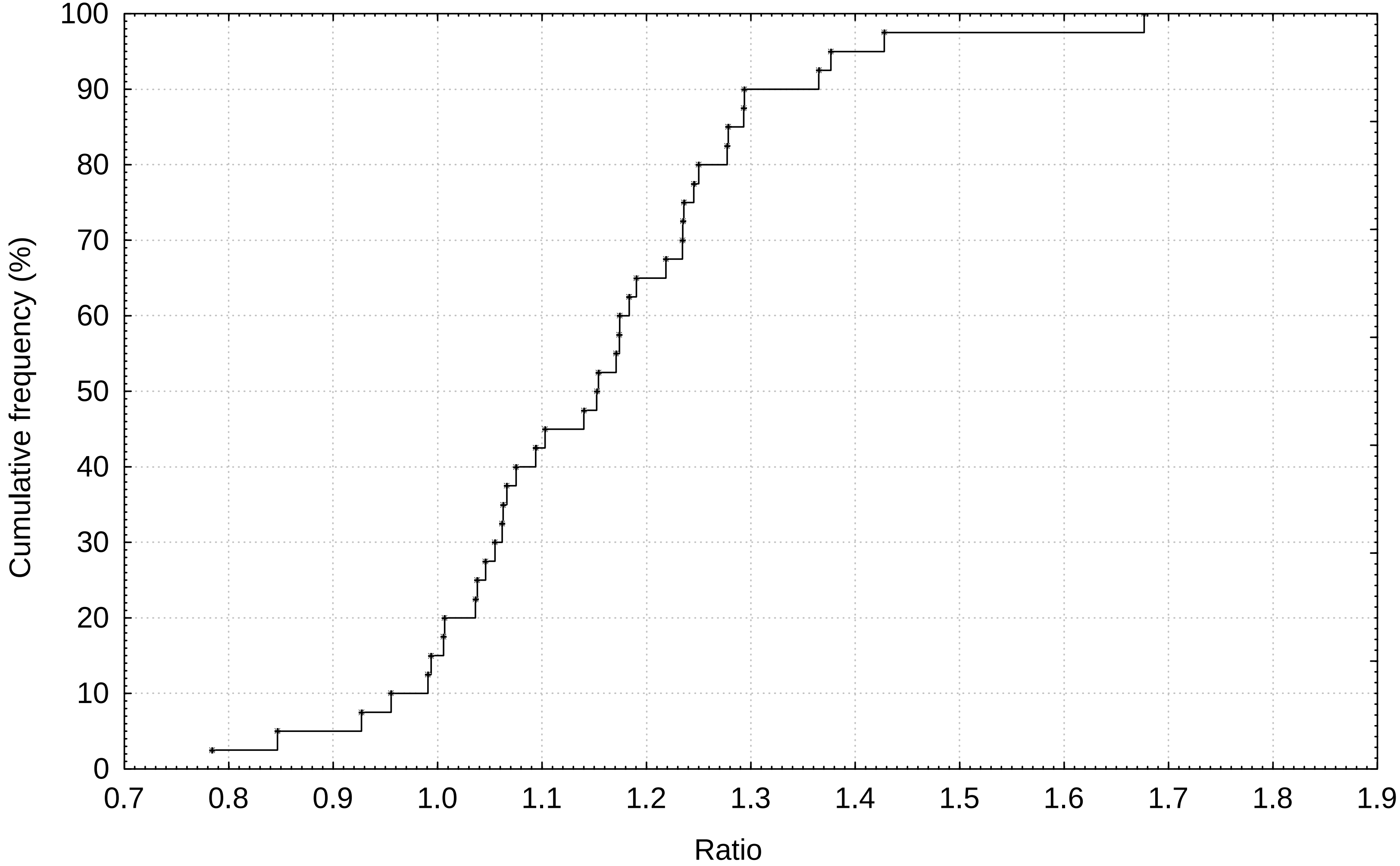

Fig 2: Probability values

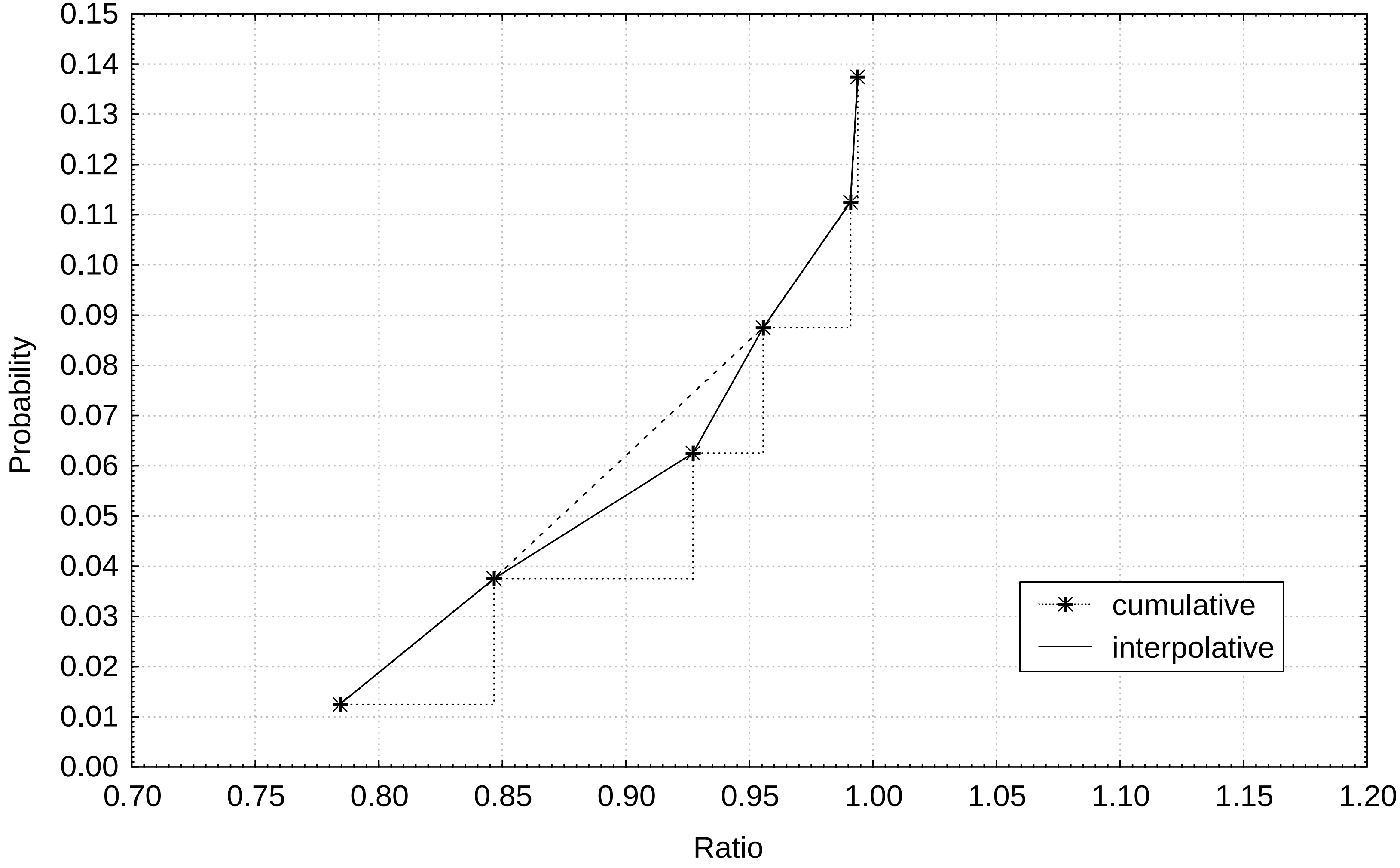

Fig 3: Probability values

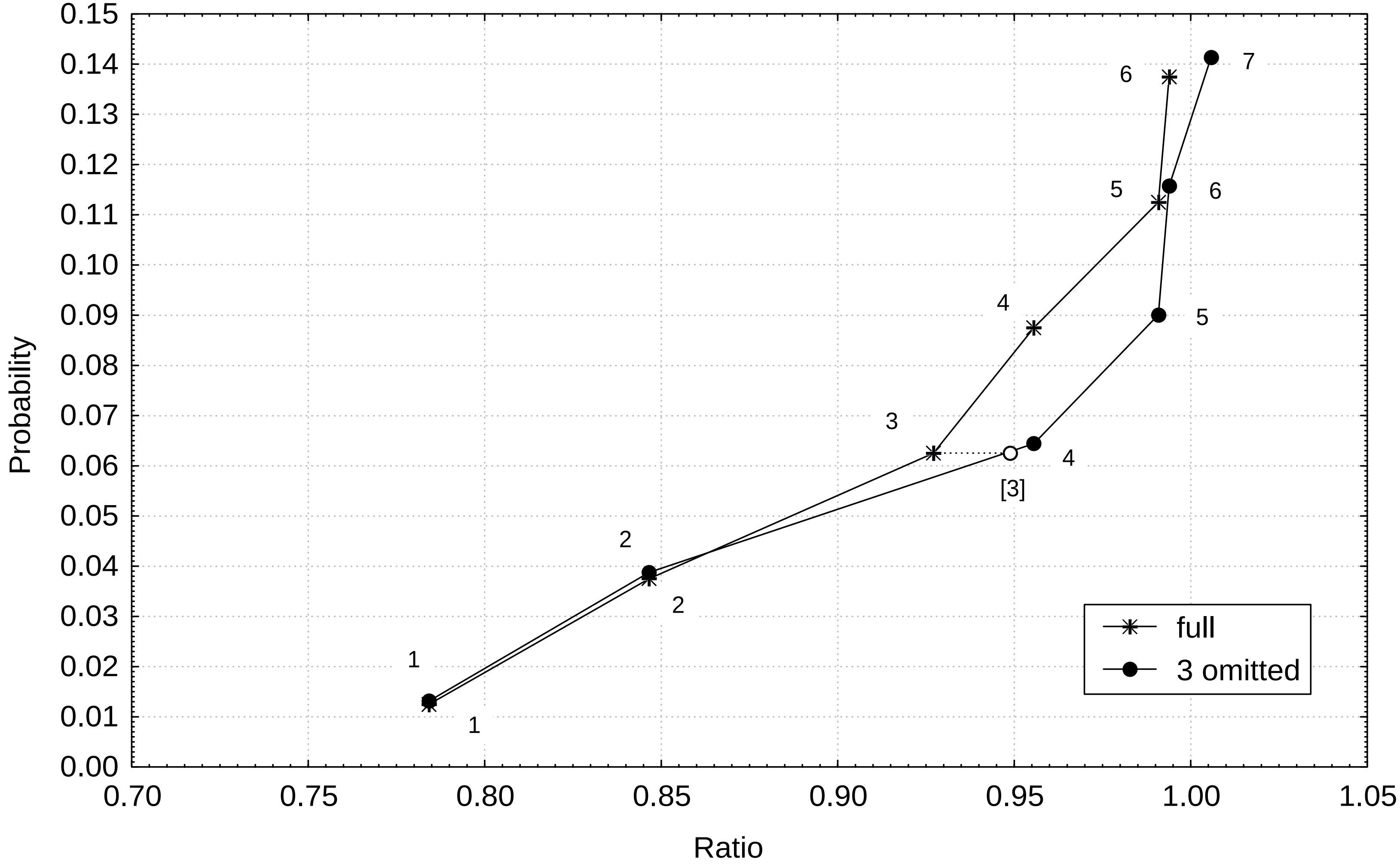

Fig 4: Cfa probabilities

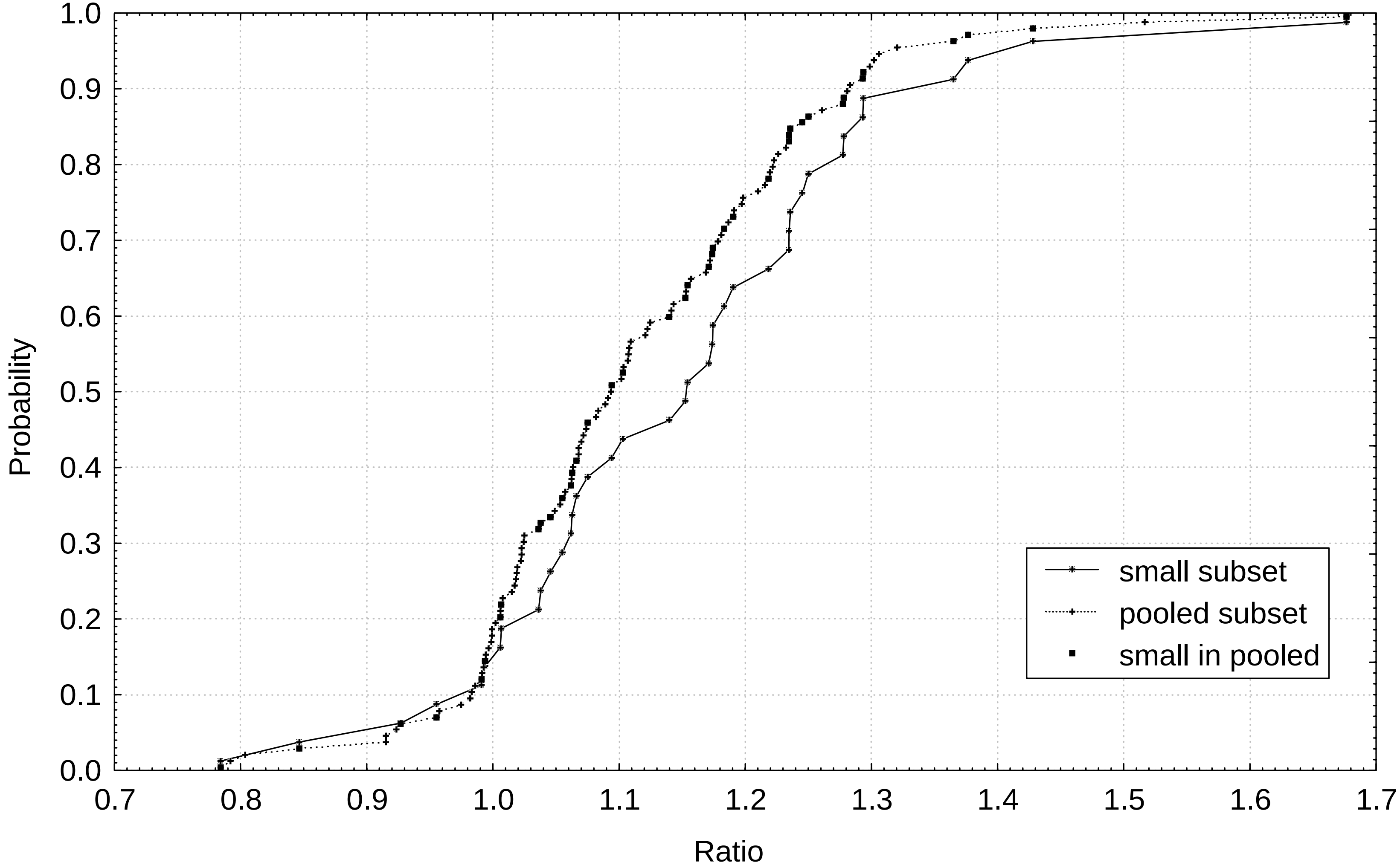

Table 1: Results for small cfa contracts subset only

Contract	R	p	$\hat{R}^X$	$R-\hat{R}^X$	$\left\|R-\hat{R}^X\right\|$
2	0.7842	0.0125	0.8446	-0.0603	0.0603
6	0.8466	0.0375	0.9218	-0.0752	0.0752
13	0.9271	0.0625	0.9488	-0.0216	0.0216
18	0.9556	0.0875	0.9853	-0.0297	0.0297
32	0.9908	0.1125	0.9896	0.0013	0.0013
35	0.9939	0.1375	1.0037	-0.0099	0.0099
48	1.0058	0.1625	1.0046	0.0011	0.0011
50	1.0067	0.1875	1.0306	-0.0238	0.0238
76	1.0363	0.2125	1.0313	0.0049	0.0049
77	1.0380	0.2375	1.0436	-0.0056	0.0056
81	1.0459	0.2625	1.0505	-0.0046	0.0046
92	1.0549	0.2875	1.0573	-0.0024	0.0024
99	1.0619	0.3125	1.0603	0.0016	0.0016
102	1.0628	0.3375	1.0648	-0.0021	0.0021
106	1.0663	0.3625	1.0706	-0.0043	0.0043
114	1.0751	0.3875	1.0833	-0.0082	0.0082
130	1.0940	0.4125	1.0915	0.0025	0.0025
138	1.1030	0.4375	1.1199	-0.0169	0.0169
161	1.1401	0.4625	1.1296	0.0105	0.0105
169	1.1524	0.4875	1.1472	0.0052	0.0052
171	1.1540	0.5125	1.1615	-0.0074	0.0074
177	1.1711	0.5375	1.1633	0.0077	0.0077
180	1.1741	0.5625	1.1725	0.0016	0.0016
181	1.1744	0.5875	1.1780	-0.0036	0.0036
186	1.1835	0.6125	1.1806	0.0030	0.0030
188	1.1903	0.6375	1.1963	-0.0059	0.0059
201	1.2186	0.6625	1.2052	0.0134	0.0134
212	1.2345	0.6875	1.2237	0.0109	0.0109
213	1.2348	0.7125	1.2349	-0.0002	0.0002
214	1.2358	0.7375	1.2375	-0.0017	0.0017
218	1.2454	0.7625	1.2392	0.0062	0.0062
220	1.2500	0.7875	1.2522	-0.0022	0.0022
224	1.2773	0.8125	1.2553	0.0220	0.0220
225	1.2783	0.8375	1.2799	-0.0016	0.0016
230	1.2931	0.8625	1.2805	0.0126	0.0126
232	1.2938	0.8875	1.3012	-0.0074	0.0074
242	1.3650	0.9125	1.3010	0.0640	0.0640
245	1.3766	0.9375	1.3689	0.0077	0.0077
249	1.4278	0.9625	1.3879	0.0399	0.0399
257	1.6767	0.9875	1.4291	0.2476	0.2476
Total	*45.8966*	*20.0000*	*45.7274*	*0.1692*	*0.7583*
Mean	*1.1474*	*0.5000*	*1.1432*	*0.0042*	*0.0190*
SD	*0.1648*	*0.2923*	*0.1328*	*0.0451*	*0.0411*

Table 2: Results for all cfa subsets

Contract	R	p	$\hat{R}^X$	$R-\hat{R}^X$	$\lvert R-\hat{R}^X \rvert$
2	0.7842	0.0041	0.7919	-0.0077	0.0077
6	0.8466	0.0289	0.9122	-0.0656	0.0656
13	0.9271	0.0620	0.9536	-0.0265	0.0265
18	0.9556	0.0702	0.9557	-0.0001	0.0001
32	0.9908	0.1198	0.9908	0.0000	0.0000
35	0.9939	0.1446	0.9940	-0.0001	0.0001
48	1.0058	0.2025	1.0050	0.0007	0.0007
50	1.0067	0.2190	1.0073	-0.0006	0.0006
76	1.0363	0.3182	1.0339	0.0024	0.0024
77	1.0380	0.3264	1.0428	-0.0048	0.0048
81	1.0459	0.3347	1.0453	0.0006	0.0006
92	1.0549	0.3595	1.0559	-0.0010	0.0010
99	1.0619	0.3760	1.0607	0.0012	0.0012
102	1.0628	0.3926	1.0631	-0.0003	0.0003
106	1.0663	0.4091	1.0660	0.0004	0.0004
114	1.0751	0.4587	1.0780	-0.0029	0.0029
130	1.0940	0.5083	1.0977	-0.0037	0.0037
138	1.1030	0.5248	1.1028	0.0002	0.0002
161	1.1401	0.5992	1.1314	0.0087	0.0087
169	1.1524	0.6240	1.1469	0.0055	0.0055
171	1.1540	0.6405	1.1547	-0.0006	0.0006
177	1.1711	0.6653	1.1700	0.0011	0.0011
180	1.1741	0.6818	1.1730	0.0011	0.0011
181	1.1744	0.6901	1.1754	-0.0010	0.0010
186	1.1835	0.7149	1.1825	0.0010	0.0010
188	1.1903	0.7314	1.1879	0.0024	0.0024
201	1.2186	0.7810	1.2167	0.0020	0.0020
212	1.2345	0.8306	1.2330	0.0016	0.0016
213	1.2348	0.8388	1.2347	0.0000	0.0000
214	1.2358	0.8471	1.2364	-0.0005	0.0005
218	1.2454	0.8554	1.2379	0.0075	0.0075
220	1.2500	0.8636	1.2474	0.0026	0.0026
224	1.2773	0.8802	1.2627	0.0146	0.0146
225	1.2783	0.8884	1.2778	0.0006	0.0006
230	1.2931	0.9132	1.2840	0.0090	0.0090
232	1.2938	0.9215	1.2935	0.0003	0.0003
242	1.3650	0.9628	1.3224	0.0426	0.0426
245	1.3766	0.9711	1.3668	0.0098	0.0098
249	1.4278	0.9793	1.3795	0.0483	0.0483
257	1.6767	0.9959	1.5172	0.1595	0.1595
Total	*45.8966*	*22.7355*	*45.6885*	*0.2081*	*0.4390*
Mean	*1.1474*	*0.5684*	*1.1422*	*0.0052*	*0.0110*
SD	*0.1648*	*0.3089*	*0.1438*	*0.0296*	*0.0279*

Table 3: Comparison of the two poolings

Contract	{1} R	{2} $\lvert R-\hat{R}^X\rvert$ - $\lvert R-\hat{R}^X\rvert$	{3} $\hat{R}(1)\lvert p(2)$	{4} Abs diff $\lvert\{1\}-\{3\}\rvert$	{5} Sum {4}+{2}
2	0.7842	-0.0527	0.7425	0.0417	-0.0109
6	0.8466	-0.0096	0.8252	0.0214	0.0118
13	0.9271	0.0048	0.9255	0.0017	0.0065
18	0.9556	-0.0296	0.9359	0.0196	-0.0099
32	0.9908	-0.0013	0.9917	0.0009	-0.0004
35	0.9939	-0.0098	0.9973	0.0034	-0.0064
48	1.0058	-0.0004	1.0244	0.0187	0.0183
50	1.0067	-0.0233	1.0367	0.0300	0.0067
76	1.0363	-0.0025	1.0621	0.0258	0.0233
77	1.0380	-0.0009	1.0624	0.0244	0.0235
81	1.0459	-0.0039	1.0627	0.0168	0.0128
92	1.0549	-0.0014	1.0659	0.0110	0.0096
99	1.0619	-0.0004	1.0711	0.0092	0.0088
102	1.0628	-0.0018	1.0790	0.0162	0.0144
106	1.0663	-0.0040	1.0914	0.0251	0.0212
114	1.0751	-0.0053	1.1344	0.0593	0.0540
130	1.0940	0.0012	1.1538	0.0597	0.0609
138	1.1030	-0.0167	1.1624	0.0594	0.0427
161	1.1401	-0.0019	1.1787	0.0386	0.0367
169	1.1524	0.0004	1.1867	0.0343	0.0346
171	1.1540	-0.0068	1.1937	0.0397	0.0328
177	1.1711	-0.0067	1.2204	0.0493	0.0427
180	1.1741	-0.0005	1.2309	0.0568	0.0563
181	1.1744	-0.0026	1.2346	0.0602	0.0576
186	1.1835	-0.0020	1.2349	0.0513	0.0494
188	1.1903	-0.0035	1.2356	0.0452	0.0417
201	1.2186	-0.0114	1.2488	0.0302	0.0187
212	1.2345	-0.0093	1.2781	0.0435	0.0342
213	1.2348	-0.0001	1.2791	0.0444	0.0442
214	1.2358	-0.0012	1.2840	0.0482	0.0470
218	1.2454	0.0013	1.2889	0.0435	0.0448
220	1.2500	0.0004	1.2931	0.0431	0.0435
224	1.2773	-0.0074	1.2936	0.0162	0.0088
225	1.2783	-0.0010	1.2964	0.0181	0.0171
230	1.2931	-0.0036	1.3653	0.0723	0.0687
232	1.2938	-0.0071	1.3692	0.0754	0.0683
242	1.3650	-0.0214	1.4309	0.0659	0.0445
245	1.3766	0.0021	1.5132	0.1365	0.1386
249	1.4278	0.0084	1.5954	0.1677	0.1760
257	1.6767	-0.0881	1.8433	0.1666	0.0785
Total	*45.8966*	*-0.3193*	*47.5189*	*1.7912*	*1.4719*
Mean	*1.1474*	*-0.0080*	*1.1880*	*0.0448*	*0.0368*
SD	*0.1648*	*0.0168*	*0.2023*	*0.0380*	*0.0362*

Table 4: Summary of poolings

Pooling	R	p	$\hat{R}^X$	$R-\hat{R}^X$	$\left\|R-\hat{R}^X\right\|$
1					
Total	45.8966	20.0000	45.7274	0.1692	0.7583
Mean	1.1474	0.5000	1.1432	0.0042	0.0190
SD	0.1648	0.2923	0.1328	0.0451	0.0411
1 2					
Total	45.8966	22.7160	45.6816	0.2150	0.4585
Mean	1.1474	0.5679	1.1420	0.0054	0.0115
SD	0.1648	0.3085	0.1438	0.0295	0.0277
1 3					
Total	45.8966	21.3875	45.6301	0.2665	0.6115
Mean	1.1474	0.5347	1.1408	0.0067	0.0153
SD	0.1648	0.3011	0.1364	0.0429	0.0406
1 2 3					
Total	45.8966	22.7355	45.6885	0.2081	0.4390
Mean	1.1474	0.5684	1.1422	0.0052	0.0110
SD	0.1648	0.3089	0.1438	0.0296	0.0279

Table 5: Summary of comparisons for small cfa contracts

Contract	{1} R	{2} $\left\|R-\hat{R}^X\right\|$ - $\left\|R-\hat{R}^X\right\|$	{3} $\hat{R}(1)\|p(2)$	{4} Abs diff $\left\|\{1\}-\{3\}\right\|$	{5} Sum {4}+{2}
Small					
Total	45.8966	0.0000	45.8966	0.0000	0.0000
Mean	1.1474	0.0000	1.1474	0.0000	0.0000
SD	0.1648	0.0000	0.1648	0.0000	0.0000
Small-medium					
Total	45.8966	-0.2998	47.2185	1.5307	1.2309
Mean	1.1474	-0.0075	1.1805	0.0383	0.0308
SD	0.1648	0.0170	0.1901	0.0239	0.0236
Small-large					
Total	45.8966	-0.1467	46.9064	1.1194	0.9726
Mean	1.1474	-0.0037	1.1727	0.0280	0.0243
SD	0.1648	0.0077	0.1946	0.0365	0.0383
Small-medium-large					
Total	45.8966	-0.3193	47.5189	1.7912	1.4719
Mean	1.1474	-0.0080	1.1880	0.0448	0.0368
SD	0.1648	0.0168	0.2023	0.0380	0.0362

Table 6: Best poolings

Subset	N	Ratio Mean	Sd	Best pooling	OC vs single subset
Construction type					
1	13	1.1855	0.2290	1	0.0000
2	1	1.2783	-	-	-
3	52	1.1242	0.1610	1 2 3 7	-0.0098
4	27	1.1587	0.2097	4	0.0000
5	70	1.1194	0.1447	5	0.0000
6	1	1.1981	-	-	-
7	79	1.1042	0.1423	7	0.0000
8	11	1.1639	0.0821	1 2 6 8	-0.0006
9	5	1.0375	0.1172	3 9	-0.0354
Total	*259*	*1.1244*	*0.1580*		
Group					
1	8	1.0194	0.1840	1	0.0000
2	0	-	-	-	-
3	121	1.1327	0.1540	3	0.0000
4	4	1.1321	0.0662	4 5 9 12	-0.0149
5	1	1.0151	-	-	-
6	0	-	-	-	-
7	0	-	-	-	-
8	1	1.0912	-	-	-
9	6	1.1248	0.0885	1 3 4 9	-0.0063
10	8	1.1241	0.1041	3 4 9 10 13	-0.0119
11	68	1.0884	0.1100	11	0.0000
12	4	1.0949	0.0858	4 8 11 12	-0.0556
13	8	1.3932	0.3759	13	0.0000
14	30	1.1181	0.1406	1 3 8 9 11-14	-0.0040
15	*0*	-	-		-
Total	*259*	*1.1244*	*0.1580*		
Number of bids					
1 Small	87	1.0902	0.1468	1	0.0000
2 Medium	80	1.1326	0.1622	2	0.0000
3 Large	92	1.1496	0.1604	1 3	0.0000
Total	*259*	*1.1244*	*0.1580*		
Nature					
1	200	1.1273	0.1490	1	0.0000
2	58	1.1169	0.1871	2	0.0000
3	1	0.9806	-	-	-
Total	*259*	*1.1244*	*0.1580*		
CFA					
1 Small	40	1.1474	0.1648	1	0.0000
2 Medium	41	1.0845	0.1193	2	0.0000
3 Large	40	1.1058	0.1206	1-3	-0.0006
Total	*121*	*1.1123*	*0.1378*		
Estimate					
1 Small	90	1.1557	0.1792	1	0.0000
2 Medium	91	1.1000	0.1173	2	0.0000
3 Large	78	1.1171	0.1689	3	0.0000
Total	*259*	*1.1244*	*0.1580*		